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The Critical Foundations

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Topic: National Math Panel: Critical Foundations for Algebra
Practice: Mathematics Preparation for Algebra

Highlights

- Brief overview of the Conceptual Knowledge and Skills Task Group
- Discussion of the critical foundations—what do students need to know and know well to be successful in algebra
- Explanation of each of the critical skills and examples
- Why it's important for students to develop fluency in whole numbers, fractions, and certain aspects of geometry and measurement
- Discussion of number sense, fractions, and using the number line
- Why automaticity with basic facts is important
- Importance of developing proficiency with fractions
- The connection between understanding concepts, computational fluency, and solving problems
- Importance of establishing a coherent progression of skill development
- Why it's important for students to understand how mathematics works

About the Interviewee

Dr. Francis (Skip) Fennell is a member of the National Mathematics Advisory Panel, Chair of the Conceptual Knowledge and Skills Task Group, and member of the National Survey of Algebra I Teachers Subcommittee and Assessment Task Group. Fennell is a mathematics educator and has experience as a classroom teacher, a principal, and a supervisor of instruction. He is a Professor of Education at McDaniel College in Westminster, MD and the immediate Past President of the National Council of Teachers of Mathematics (NCTM). Widely published in professional journals and textbooks related to elementary and middle-grade mathematics education, Dr. Fennell has also authored chapters in yearbooks and resource books published by the National Council of Teachers of Mathematics. In addition, he has played key leadership roles with NCTM, the Research Council for Mathematics Learning, the Mathematical Sciences Education Board, the National Science Foundation, the Maryland Mathematics Commission, the United States National Commission for Mathematics Instruction, and the Association for Mathematics Teacher Educators.

Dr. Fennell has received numerous honors and awards, including Maryland's Outstanding Mathematics Educator (1990), McDaniel College's Professor of the Year (1997), the Glenn Gilbert National Leadership Award from the National Council of Supervisors of Mathematics, and the CASE - Carnegie Foundation Professor of the Year - Maryland (1997). He has also been the principal investigator on grants from the National Science Foundation, the U.S. Department of Education, the Maryland Higher Education Commission, and the ExxonMobil Foundation.

Full Transcript

My name is Francis (Skip) Fennell. I am Professor of Education at McDaniel College in Westminster, Maryland.

I served as a member of the National Mathematics Advisory Panel, chairing the Conceptual Knowledge & Skills Task Group. Also served as a member of the Assessment Task Group and also served as a member of a subcommittee that worked on the teacher survey of over 700 teachers of algebra in this country.

The Conceptual Knowledge & Skills Task Group which I chaired was essentially the math piece of this work, and it was our job to take a look at the President's message, which says, essentially: What do students need to know and know well before they will have success in algebra? What we did was look at the elements of all that kids experience at the elementary school level and on into middle school in terms of mathematics curriculum and said, these are the must-haves. In other words, you must have a full understanding and proficiency with everything there is to know about whole numbers, and by that I mean, it begins with counting, place value, working with all the operations—addition, subtraction, multiplication, and division—and the basic facts, retrieval of those basic facts, being able to use a standard algorithm successfully.

Another huge piece for us—and it popped up time and time and again in our work—was work with fractions, and so we said very, very loudly across numerous task groups, but particularly ours, that you need to know all about fractions, and you need to know all about decimals, and you need to know all about percent, and how that, then, leads to work in working with ratio and proportion, which are building blocks to algebra. And so, similarly to the kinds of background that kids ought to have with whole numbers, you ought to know what fractions are, you need to know how to work with them. You need to be able to determine their size and be able to work with equivalent areas of fractions that are comfortable in our surroundings like decimals, like percent. If it's 2% milk, what does that look like as a decimal? What does that look like as a fraction? So, work with whole numbers, success with all facets of working with fractions—defined here as fractions, decimals, and percent. And also particular aspects of measurement and geometry, things like two-dimensional shape, three-dimensional shape, perimeter, area, volume, surface area, work with similar triangle—those were our must-haves. We call those our critical foundations.

Children come to school with this sort of inherent and then experience-laden sense of number that, frankly, we don't take advantage of as much. Now, having said that, there are also things we do in the classroom and schools that kind of build that sense of number. One of those is sort of looking at number relative to magnitude: "You said you went to the game last night. I wonder, how big was the crowd? Was it the same kind of crowd that we might have in our school auditorium? More? Less? How do you know?" Sense of magnitude, very experience-driven. And so this connects to measurement: "About how far do you think it is from here to the principal's office? Let's estimate that." So, connecting to estimation through issues of magnitude. And so this notion of using estimation, using some mental math, and then connecting that to the structure of mathematics, in this case, the distributive property, helps build that sense of number.

And it is so important…I mean, I can't emphasize enough that as we think about these critical foundations involving whole number, involving fractions, that number sense is the element of that that makes it make sense. That seeing numbers on a number line, that knowing, for instance—I will give you a good example. I was working with some kids in fractions and we were using number lines and we were comparing fractions on the number line. And now we were sort of between zero and one, and I said, "Okay, where would you put 9/5ths?" And this kid looked at me with as much seriousness as any 10-year-old can possibly commands it, "Can't do it." I said, "What do you mean you can't do it?" "Well, there is no place to put it." And it occurred to me that in their work with thinking about how fractions can go on a number line, they were only between zero and one, and that here is a number, 9/5ths, that's out there somewhere. So comfort, understanding how things can compare, where to place them and so forth—very experience-driven. It needs to sort of undergird everything we do with working with whole numbers and fractions, if we really want these critical foundations to make sense.

As we work with particular operations—initially addition, subtraction, and then multiplication, division—there are these subset of elements of both those two sets of operations known as the basic facts. And so being able to recall 6 + 6, being able to recall 6 x 7, are integral, if you will, building blocks to then working toward algorithms. And so the notion of automaticity with basic facts is really something that will allow students to move forward faster as they work with algorithms. As they get more connected to how these operations work, we then want to provide access to a standard algorithm. But the concern we have, and historically, we have had, is that while it's important that students have access to algorithms, while it's important that students have automaticity with their facts, there needs to be this connection to understanding how those things work. It doesn't make any sense for kids to have access to a standard algorithm for 13 x 7 if they don't get how it works.

This whole issue about rational numbers—and when we think about rational numbers, by the way, that's both positive and negative fractions and everything that kind of connects that—but for classroom teachers, for people at home, we are talking about fractions: *a* over *b*, 5/6ths, 2/3rds kinds of things, but we are also talking about decimals, because decimals are a way in which we represent fractions. We are also talking about percent, and how that important mathematics build to work with ratio and proportions and so forth. And that piece is so critical to algebra because in many ways that's how we use, you know, how what people refer to as arithmetic builds into higher-level mathematics.

One thing, I think, we have to do differently, and that is to think about—I would almost want to coin the phrase "fraction sense"—I mean, we talk a lot about number sense, but how that played out in school far too frequently only exists as a subset of number sense; thinking about whole numbers only. We must think about that as those other numbers; that knowing that 2/3rds is less than 7/8ths, and knowing that, frankly, 9/5ths can, in fact, be a fraction, it just happens to exist greater than one. All of those things, those kind of background experiences—and this is key—before they get to procedures for adding, subtracting, multiplying, and dividing fractions. And we get to those, but I submit that far too many students get procedures for doing that stuff without really understanding how fractions work.

There are elements of the measurement and geometry piece that popped. Comfort with, understanding of two-dimensional shape and then how we think about that, because if I look at a triangle, at some point, not only will I need to describe that, but I'll want to measure it. By that I mean, what's the distance around with the perimeter of that figure, what's the area of that figure? So those common measurements linked to geometric shape, also link to another important notion, and that is, as I think about proportion and I think about ratio, I might be looking at a size of a figure that I might want to blow up, times four. And so, here on that figure is a triangle and now I have a similar triangle except it's four times larger. And so, here's the proportion, 4:1, and that's going to be part of a prerequisite that many, many students are going to have as they engage in this mathematics called algebra. So those, again, are some of the building blocks that tie into our critical foundations.